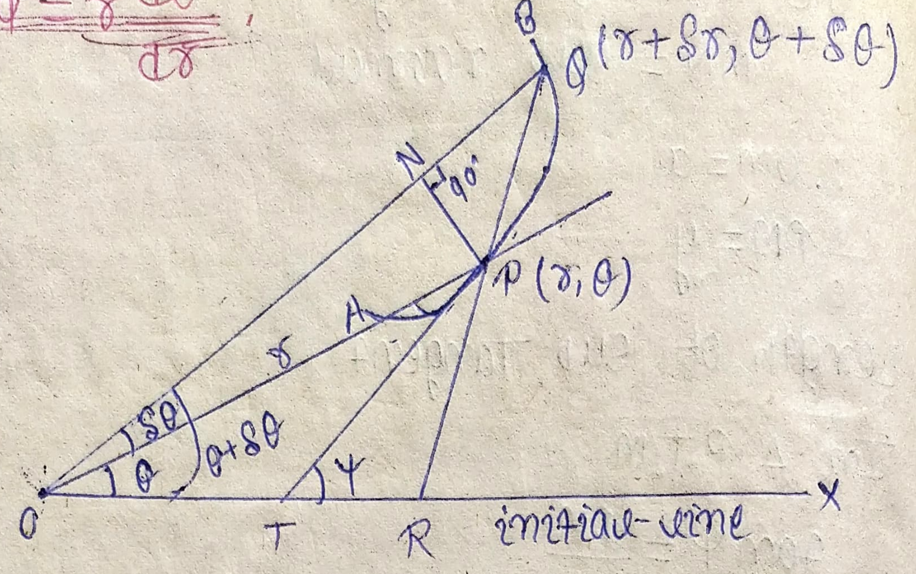
Q No. → Prove that:-

(i) $$\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$$

(ii) $$\frac{ds}{dr} = \sqrt{1 + \left(r\frac{d\theta}{dr}\right)^2}$$

(iii) $\tan\phi = r\dfrac{d\theta}{dr}$



Let O = Pole

∴ OX = initial - line

Let A be a fixed point on the curve

Let P(r, θ) be co-ordinate of any point P on the curve such that

$$AP = s$$

Let Q be another point $(r+\delta r,\ \theta+\delta\theta)$ very close to P

Let $AQ = s + \delta s$

Length of cu $PQ = \cancel{s} + \delta s - \cancel{s} = \delta s$

from P draw $PN \perp OQ$

In Δ OPN

$\sin\delta\theta = \dfrac{PN}{r}$ ∴ $PN = r\sin\delta\theta$

$\cos\delta\theta = \dfrac{ON}{r}$ ∴ $ON = r\cos\delta\theta$

Now in ΔPNQ

$\because PQ^2 = PN^2 + NQ^2$

$= (r\sin\delta\theta)^2 + (OQ - ON)^2$

$= r^2\sin^2\delta\theta + (r + \delta r - r\cos\delta\theta)^2$

$= r^2\sin^2\delta\theta + [r(1-\cos\delta\theta) + \delta r]^2$

$PQ^2 = r^2\sin^2\delta\theta + \left[r \times 2\sin^2\frac{\delta\theta}{2} + \delta r\right]^2$

Dividing by $\delta\theta^2$

or, $\dfrac{PQ^2}{\delta\theta^2} = \dfrac{r^2\sin^2\delta\theta}{(\delta\theta)^2} + \left[\dfrac{2r\sin^2\frac{\delta\theta}{2} + \delta r}{\delta\theta^2}\right]^2$

or, $\dfrac{PQ^2}{\delta s^2} \times \dfrac{\delta s^2}{\delta\theta^2} = r^2\left(\dfrac{\sin\delta\theta}{\delta\theta}\right)^2 + \left[\dfrac{2r\sin^2\frac{\delta\theta}{2}}{\delta\theta} + \dfrac{\delta r}{\delta\theta}\right]^2$

or, $\left(\dfrac{\delta s}{\delta\theta}\right)^2 = r^2\left(\dfrac{\sin\delta\theta}{\delta\theta}\right)^2 + \left[\cancel{2}r\left[\dfrac{\sin\frac{\delta\theta}{2}}{\frac{\delta\theta}{2}}\right] \times \cancel{2} \times \sin\frac{\delta\theta}{2} + \dfrac{\delta r}{\delta\theta}\right]^2$

when $Q \to P$

then $\delta\theta \to 0$

Hence on taking limit, we from

$\left(\dfrac{ds}{d\theta}\right)^2 = r^2 + \left(\dfrac{dr}{d\theta}\right)^2$

$\dfrac{ds}{d\theta} = \sqrt{r^2 + \left(\dfrac{dr}{d\theta}\right)^2}$ — (1)

proved.

[(ii) QN. में भी (i) वाला शुरू से पूरा लिखना है, तथा जब (i) QN. आएगा तब सभी (ii) केवल नहीं लिखना है]

(ii)

} } }

Now multiplying both sides by $\dfrac{d\theta}{dr}$

$$\frac{ds}{\cancel{d\theta}} \times \frac{d\theta}{dr} = \frac{d\theta}{dr}\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$$

$$\frac{ds}{dr} = \sqrt{r^2\left(\frac{d\theta}{dr}\right)^2 + \cancel{\left(\frac{d\theta}{dr}\right)^2 \times \left(\frac{dr}{d\theta}\right)^2}}$$

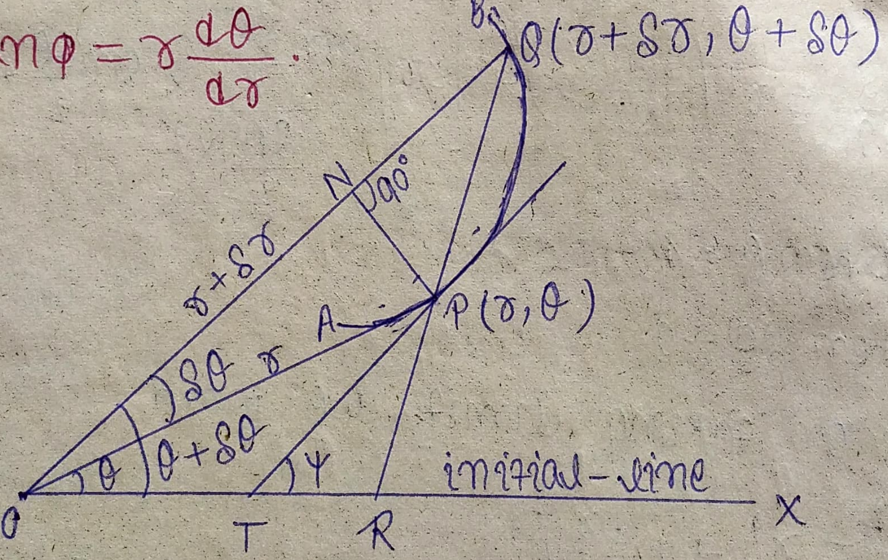
$$= \sqrt{r^2\left(\frac{d\theta}{dr}\right)^2 + 1}$$

$$\frac{ds}{dr} = \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}$$

$$= \sqrt{1 + \left(r\frac{d\theta}{dr}\right)^2}$$

(iii) Prove that :-

(i) $\tan\phi = r\dfrac{d\theta}{dr}$.



Ans. → Let O = Pole

OX = initial - line

Let ϕ be the angle between the tangent at the point $P(r, \theta)$ and the radius vector $OP = r$ of the curve $r = f(\theta)$

In $\Delta\ OPN$

$\because \sin\delta\theta = \frac{PN}{r} \therefore PN = r\sin\delta\theta$

$\cos\delta\theta = \frac{ON}{r} \therefore ON = r\cos\delta\theta$

Now, $QN = OQ - ON$

$= r + \delta r - r\cos\delta\theta$

$= r(1-\cos\delta\theta) + \delta r$

$QN = r \times 2\sin^2\frac{\delta\theta}{2} + \delta r$

In ΔPQN

Now, $\tan PQN = \frac{PN}{QN}$

$$= \frac{r\sin\delta\theta}{\delta r + 2r\sin^2\frac{\delta\theta}{2}}$$

when $Q \to P$

$\delta\theta \to 0$

$\therefore \angle PQN = \angle OPR = \phi$

$$\tan\phi = \frac{\frac{r\sin\delta\theta}{\delta\theta}}{\underset{\delta\theta\to 0}{Lt}\ \frac{\delta r}{\delta\theta} + \frac{2r\sin^2\frac{\delta\theta}{2}}{\delta\theta}}$$

$$\tan\phi = \underset{\delta\theta\to 0}{Lt}\ \frac{r\left(\frac{\sin\delta\theta}{\delta\theta}\right)}{\left(\frac{\delta r}{\delta\theta}\right) + \cancel{2}r\left(\frac{\sin\frac{\delta\theta}{2}}{\frac{\delta\theta}{2}}\right) \times \cancel{2} \times \sin\frac{\delta\theta}{2}}$$

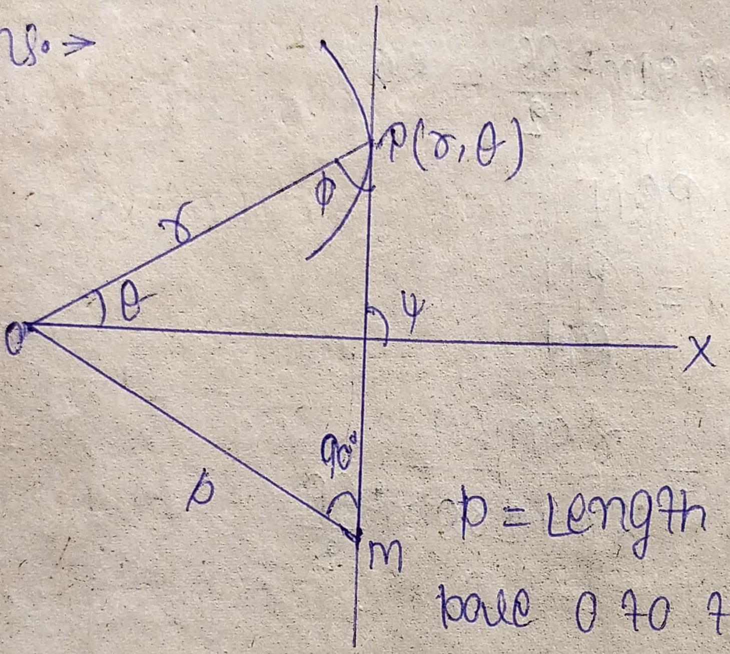
or, $$\tan\phi = \frac{r \times 1}{\frac{dr}{d\theta} + r \times 1 \times 0}$$

$$\tan\phi = \frac{r}{\frac{dr}{d\theta}} = r\frac{d\theta}{dr}$$ proved.

QNo.→ Prove that

(i) $p = r\sin\phi$.

Ans.→



Let $r = f(\theta)$ be the eqn. of the curve.

$P(r,\theta)$ be any point on the curve

p = Length of ⊥ from the pole O to the tangent of the curve at the point (r,θ)

ϕ = angle between the tangent at $P(r,\theta)$ and ~~r~~ the radius vector

So, $OP = r$

Now, in ΔOPM

$$\sin\phi = \frac{OM}{OP}$$

$$\text{or, } \sin\phi = \frac{p}{r}$$

$$\therefore p = r\sin\phi.$$

QNo.→ Prove that the equation of the normal to the curve, $y=f(x)$ at the point (x,y) is $(X-x)+(Y-y)\frac{dy}{dx}=0$ ~~and~~

Ans.→ we know that normal is a st. line $\perp$ to the tangent at the point of contact.

∵ eqn. of tangent at the pt. (x,y) to the curve $y=f(x)$ is given,

$$Y-y=\frac{dy}{dx}(X-x) \quad — ①$$

slope of tangent $m=\frac{dy}{dx}$

Hence the slope of the normal

$$=-\frac{1}{\frac{dy}{dx}}$$

Hence the eqn. of normal is give by

$$Y-y=-\frac{1}{\frac{dy}{dx}}(X-x)$$

or, $(Y-y)\frac{dy}{dx}=-(X-x)$

or, $(X-x)+(Y-y)\frac{dy}{dx}=0$

is the eqn. of normal at the point (x,y) to the curve $y=f(x)$.

QN.→ Prove that the equation of the normal to the curve $f(x,y)=0$ at the point (x,y) is

$$\frac{X-x}{f_x}=\frac{Y-y}{f_y}.$$

Ans.→ We know that normal is a St. line ⊥ to the tangent at the point of contact.

∵ The eqn. of tangent at the point (x,y) to the curve $f(x,y)=0$ is

$$(X-x)f_x+(Y-y)f_y=0$$

$$\text{or, } Y-y=-\frac{f_x}{f_y}(X-x)$$

Hence the eqn. of normal to the curve $f(x,y)=0$ at the point (x,y) is

$$Y-y=\frac{1}{\frac{f_x}{f_y}}(X-x)$$

$$\text{or, } Y-y=\frac{f_y}{f_x}(X-x)$$

$$\text{or, } \frac{Y-y}{f_y}=\frac{X-x}{f_x}$$

is the reqd. eqn. of normal to the curve $f(x,y)=0$.

The - End